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Rudolf
On the Bakshali manuscript

The Hon'ble
Sir Alfred Croft
with the author's
Compliments.

ON THE

BAKHSHĀḶĪ MANUSCRIPT.

BY

Dr. R. HOERNLE.

WITH THREE PHOTOZINCOGRAPHS.

VIENNA
ALFRED HÖLDER
EDITOR OF THE COURT AND OF THE UNIVERSITY
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The manuscript which I have the honour, this morning, of placing before you, was found, as you will recollect, in May 1881, near a village called Bakhshālī, lying in the Yusufzāī district of the Peshawer division, at the extreme Northwestern frontier of India. It was dug out by a peasant in a ruined enclosure, where it lay between stones. After the find it was at once forwarded to the Lieutenant Governor of the Panjab who transmitted it to me for examination and eventual publication.

The manuscript is written in Shārada character of a rather ancient type, and on leaves of birch-bark which from age have become dry like tinder and extremely fragile. Unfortunately, probably through the careless handling of the finder, it is now in an excessively mutilated condition, both with regard to the size and the number of the leaves. Their present size, as you observe (see Plate I), is about 6 by $3^1/_2$ inches; their original size, however, must have been about 7 by $8^1/_4$ inches. This might have been presumed from the well-known fact that the old birck-bark manuscripts were always written on leaves of a squarish size. But I was enabled to determine the point by a curious fact. The mutilated leaf which contains a portion of the $27^1/_2$ sūtra, shows at top and bottom the remainders of two large square

figures, such as are used in writing arithmetical notations. These when completed prove that the leaf in its original state must have measured approximately 7 by $8^1/_4$ inches. The number of the existing leaves is seventy. This can only be a small portion of the whole manuscript. For neither beginning nor end is preserved; nor are some leaves forthcoming which are specifically referred to in the existing fragments.[1]) From all appearances, it must have been a large work, perhaps divided in chapters or sections. The existing leaves include only the middle portion of the work or of a division of it. The earliest sūtra that I have found is the 9th; the latest is the 57th. The lateral margins which usually exhibit the numbering of the leaves are broken off. It is thus impossible even to guess what the original number of the leaves may have been.

The leaves of the manuscript, when received by me, were found to be in great confusion. Considering that of each leaf the top and bottom (nearly two thirds of the whole leaf) are lost, thus destroying their connection with one another, it may be imagined that it was no easy task to read and arrange in order the fragments. After much trouble I have read and transcribed the whole, and have even succeeded in arranging in consecutive order a not inconsiderable portion of the leaves containing eighteen sūtras. The latter portion I have also translated in English.

The beginning and end of the manuscript being lost, both the name of the work and of its author are unknown. The subject of the work, however, is arithmetic. It contains a great variety of problems relating to daily life. The following are examples. ‚In a carriage, instead of 10 horses, there are yoked 5; the distance traversed by the former was one hundred, how much will the other horses be able to accomplish?‘ Te following is more complicated: ‚A certain person travels 5 yojanas on the

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[1]) Thus at the end of the 10th sūtra, instead of the usual explanation, there is the following note: *evaṁ sūtraṁ | dvitīya patre vivaritāsti.* The leaf referred to is not preserved.

first day, and 3 more on each succeeding day; another who travels 7 yojanas on each day, has a start of 5 days; in what time will they meet?' The following is still more complicated: 'Of 3 merchants the first possesses 7 horses, the second 9 ponies, the third 10 camels; each of them gives away 3 animals to be equally distributed among themselves; the result is that the value of their respective properties becomes equal; how much was the value of each merchant's original property, and what was the value of each animal?' The method prescribed in the rules for the solution of these problems is extremely mechanical, and reduces the labour of thinking to a minimum. For example, the last mentioned problem is solved thus: 'Subtract the gift (3) severally from the original quantities (7, 9, 10). Multiply the remainders (4, 6, 7) among themselves (168, 168, 168). Divide each of these products by the corresponding remainder ($\frac{168}{4}$, $\frac{168}{6}$, $\frac{168}{7}$). The results (42, 28, 24) are the values of the 3 classes of animals. Being multiplied with the numbers of the animals originally possessed by the merchants (42 . 7, 28 . 9, 24 . 10), we obtain the values of their original properties (294, 252, 240). The value of the properts of each merchant after the gift is equal (262, 262, 262).' The rules are expressed in very concise language, but are fully explained by means of examples. Generally there are two examples to each rule (or sūtra), but sometimes there are many; the 25th sūtra has no less than 15 examples. The rules and examples are written in verse; the explanations, solutions and all the rest are in prose. The metre used is the *shloka*.

The subject-matter is divided in *sūtras*. In each sūtra the matter is arranged as follows. First comes the rule, and then the example, introduced by the word *tadā*. Next, the example is repeated in the form of a notation in figures, which is called *sthāpana*. This is followed by the solution which is called *karaṇa*. Finally comes the proof, called *pratyaya*. This arrangement and terminology differ somewhat from those used in the arithmetic of Brahmagupta and Bhāskara. Instead of simply *sūtra*, the latter use the term *karaṇa-sūtra*. The example they call *uddeshaka* or *udāharaṇa*. For *sthāpana* they say *nyāsa*. As a rule they give

no full solution or proof, but the mere answer to the problem. Occasionally a solution is given, but it is not called *karaṇa.*

The system of notation used in the Bakhshālī arithmetic is much the same as that employed in the arithmetical works of Brahmagupta and Bhāskara. There is, however, a very important exception. The sign for the negative quantity is a cross (+). It looks exactly like our modern sign for the positive quantity, but is placed after the number which it qualifies. Thus $\begin{smallmatrix}12\\1\end{smallmatrix}\ \begin{smallmatrix}7\\1\end{smallmatrix}{+}$ means 12—7 (i. e. 5). This is a sign which I have not met with in any other Indian arithmetic; nor so far as I have been able to ascertain, is it known in India at all. The sign now used is a dot placed over the number to which it refers. Here, therefore, there appears to be a mark of great antiquity. As to its origin I am unable to suggest any satisfactory explanation. I have been informed by Dr. Thibaut of Benares, that Diophantus in his Greek arithmetic uses the letter ψ (short for λεῖψις) reversed (thus ψ), to indicate the negative quantity. There is undoubtedly a slight resemblance between the two signs; but considering that the Hindūs did not get their elements of the arithmetical science from the Greeks, a native origin of the negative sign seems more probable. It is not uncommon in Indian arithmetic to indicate a particular factum by the initial syllable of a word of that import subjoined to the terms which compose it. Thus addition may be indicated by *yu* (short for *yuta*), e. q. $\begin{smallmatrix}5\\1\end{smallmatrix}\ \begin{smallmatrix}7\\1\end{smallmatrix}\ ^{yu}$ means 5 + 7 (c. e. 12). In the case of substraction or the negative quantity *ṛiṇa* would be the indicatory word and *ṛi* the indicatory syllable. The difficulty is to explain the connection between the letter *ṛi* (ऋ) and the symbol +. The latter very closely resembles the letter *k* (क) in its ancient shape (+) as used in the Ashoka alphabet. The word *kaṇa* or *kaṇīyas* which had once occurred to me, is hardly satisfactory.

A whole number, when it occurs in an arithmetical operation, as may be seen from the above given examples, is indicated by placing the number 1 under it. This, however, is a practice which is still occasionally observed in India. It may be worth noting that the number one is always designated by

the word *rūpa*;[1]) thus *sarūpa* or *rūpādhika* 'adding one', *rūpoṇa* 'deducting one'. The only other instance of the use of a symbolic numeral word is the word *rasa* for six which occurs once in an example in sūtra 53.

The following statement, from the first example of the 25th Sūtra, affords a good example of the system of notation employed in the Bakhshālī arithmetic:

$$\left|\begin{array}{ccccc} & 1 & 1 & 1 & \\ \cdot & 1 & 1 & 1 & bh\bar{a}\ 32 \\ 1 & 3+ & 3+ & 3+ & \end{array}\right|\ phala\dot{m}\ 108$$

Here the initial dot is used very much in the same way as we use the letter x to denote the unknown quantity the value of which is sought. The number 1 under the dot is the sign of the whole (in this case, unknown) number. A fraction is denoted by placing one number under the other without any line of separation; thus $\begin{smallmatrix}1\\3\end{smallmatrix}$ is $\frac{1}{3}$, i. e. one-third. A mixed number is shown by placing the three numbers under one another; thus $\begin{smallmatrix}1\\1\\3\end{smallmatrix}$ is $1+\frac{1}{3}$ or $1\frac{1}{3}$, i. e. one and one-third. Hence $\begin{smallmatrix}1\\1\\3+\end{smallmatrix}$ means $1-\frac{1}{3}$ (i. e. $\frac{2}{3}$). Multiplication is usually indicated by placing the numbers side by side; thus $\left|\begin{smallmatrix}5 & 32\\8 & 1\end{smallmatrix}\right|$ *phalaṁ* 20 means $\frac{5}{8}\times 32 = 20$. Similarly $\begin{smallmatrix}1\\1\\3+\end{smallmatrix}\begin{smallmatrix}1\\1\\3+\end{smallmatrix}\begin{smallmatrix}1\\1\\3+\end{smallmatrix}$ means $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$ or $(\frac{2}{3})^3$, c. e. $\frac{8}{27}$. *Bhā* is an abbreviation of *bhāga* 'part' and means that the number preceding it is to be divided. Hence $\begin{smallmatrix}1\\1\\3+\end{smallmatrix}\begin{smallmatrix}1\\1\\3+\end{smallmatrix}\begin{smallmatrix}1\\1\\3+\end{smallmatrix}$ *bhā* means $\frac{27}{8}$. The whole statement, therefore,

$$\left|\begin{array}{ccccc} & 1 & 1 & 1 & \\ \cdot & 1 & 1 & 1 & \\ 1 & 3+ & 3+ & 3+ & bh\bar{a}\ 32 \end{array}\right|\ phala\dot{m}\ 108$$

means $\frac{27}{8}\times 32 = 108$, and may be thus explained: 'a certain number is found by dividing with $\frac{8}{27}$ and multiplying with 32; that number is 108'.

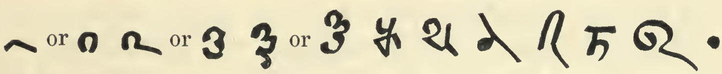
The dot is also used for another purpose, namely as one of the ten fundamental figures of the decimal system of notation

[1]) This word was at first read by me *ūpa*. The reading *rūpa* was suggested to me by Professor A. Weber, and though not so well agreeing with the manuscript characters, is probably the correct one.

or the zero (0 1 2 3 4 5 6 7 8 9). It is still so used in India for both purposes, to indicate the unknown quantity as well as the naught. With us the dot, or rather its substitute the circle (∘), has only retained the latter of its two intents, being simply the zero figure, or the 'mark of position' in the decimal system. The Indian usage, however, seems to show, how the zero arose and that it arose in India. The Indian dot, unlike our modern zero, is not properly a numerical figure at all. It is simply a sign to indicate an empty place or a hiatus. This is clearly shown by its name *shūnya* 'empty'. The empty place in an arithmetical statement might or might not be capable of being filled up, according to circumstances. Occurring in a row of figures arranged decimally or according to the 'value of position', the empty place could not be filled up, and the dot therefore signified 'naught', or stood in the place of the zero. Thus the two figures 3 and 7, placed in juxtaposition (37) mean 'thirty seven', but with an 'empty space' interposed between them (3 7), they mean 'three hundred and seven'. To prevent misunderstanding the presence of the 'empty space' was indicated by a dot (3·7), or by what in now the zero (307). On the other hand, occurring in the statement of a problem, the 'empty place' could be filled up, and here the dot which marked its presence, signified a 'something' which was to be discovered and to be put in the empty place. In the course of time, and out of India, the latter signification of the dot was discarded; and the dot thus became simply the sign for 'naught' or the zero, and assumed the value of a proper figure of the decimal system of notation, being the 'mark of position'. In its double signification which still survives in India, we can still discern an indication of that country as its birth place.

Regarding the age of the manuscript am unable to offer a very definite opinion. The composition of a Hindū work on arithmetic, such as that contained in the Bakhshālī MS. seems necessarily to presuppose a country and a period in which Hindū civilisation and Brahmanical learning flourished. Now the country in which Bakhshālī lies and which formed part of the Hindū

kingdom of Kabul, was early lost to Hindū civilisation through the conquests of the Muhammedan rulers of Ghazni, and especially through the celebrated expeditions of Mahmūd, towards the end of the 10th and the beginning of the 11th centuries A. D. In those troublons times it was a common practice for the learned Hindūs to bury their manuscript treasures. Possibly the Bakhshālī MS. may be one of these. In any case it cannot well be placed much later than the 10th century A. D. It is quite possible that it may be somewhat older. The Shārada characters used in it, exhibit in several respects a rather archaic type, and afford some ground for thinking that the manuscript may perhaps go back to the 8th or 9th century. But in the present state of our epigraphical knowledge, arguments of this kind are always somewhat hazardous. The usual form, in which the numeral figures occur in the manuscript are the following:



1 2 3 4 5 6 7 8 9 0

Quite distinct from the question of the age of the manuscript is that of the age of the work contained in it. There is every reason to believe that the Bakhshālī arithmetic is of a very considerably earlier date than the manuscript in which it has come down to us. I am disposed to believe that the composition of the former must be referred to the earliest centuries of our era, and that it may date from the 3d or 4th century A. D. The arguments making for this conclusion are briefly the following.

In the first place, it appears that the earliest mathematical works of the Hindūs were written in the *Shloka* measure;[1] but from about the end of the 5th century A. D. it became the fashion to use the Ārya measure. Āryabhaṭṭa c. 500 A. D., Varāha Mihira c. 550, Brahmagupta c. 630, all wrote in the latter measure. Not only were new works written in it, but also Shloka works were revised and recast in it. Now the Bakhshālī arith-

[1]) See Professor Kern's Introduction to Varāha Mihira.

metic is written in the Shloka measure; and this circumstance carries its composition back to a time anterior to that change of literary fashion in the 5th century A. D.

In the second place, the Bakhshālī arithmetic is written in that peculiar language which used to be called the 'Gāthā dialect', but which is rather the literary form of the ancient Northwestern Prākrit (or Pāli). It exhibits a strange mixture of what we should now call Sanskrit and Prākrit forms. As shown by the inscription (e. g., of the Indoscythian kings in Mathurā) of that period, it appears to have been in general use, in Northwestern India, for literary purposes till about the end of the 3d century A. D., when the proper Sanskrit, hitherto the language of the Brahmanic schools, gradually came into general use also for secular compositions. The older literary language may have lingered on some time longer among the Buddhists and Jains, but this would only have been so in the case of religious, not of secular compositions. Its use, therefore, in the Bakhshālī arithmetic points to a date not later than the 3d or 4th century A. D. for the composition of that work.

In the third place, in several examples, the two words *dīnāra* and *dramma* occur as denominations of money. These words are the Indian forms of the latin *denarius* and the greek *drachme*. The former, as current in India, was a gold coin, the latter a silver coin. Golden *denarii* were first coined at Rome in 207 B. C. The Indian gold pieces, corresponding in weight to the Roman gold *denarius*, were those coined by the Indoscythian kings, whose line beginning with Kadphises, about the middle of the 1st century B. C., probably extended to about the end of the 3d century A. D. Roman gold *denarii* themselves, as shown by the numerous finds, were by no means uncommon in India, in the earliest centuries of our era. The gold dīnārs most numerously found are those of the Indoscythian kings Kanishka and Huvishka, and of the Roman emperors Trajan, Hadrian and Antonius Pius, all of whom reigned in the 2nd century A. D. The way in which the two terms are used in the Bakhshālī arithmetic seems to indicate that the gold *dīnāra* and the silver

dramma formed the ordinary currency of the day. This circumstance again points to some time within the three first centuries of the Christian era as the date of its composition.

A fourth point, also indicative of antiquity which I have already adverted to, is the peculiar use of the cross (+) as the sign of the negative quantity.

There is another point which may be worth mentioning though I do not know whether it may help in determining the probable date of the work. The year is reckoned in the Bakhshālī arithmetic as consisting of 360 days. Thus in one place the following calculation is given: 'If in $\frac{800}{727}$ of a year 2982 $\frac{486}{727}$ is spent, how much is spent in one day?' Here it is explained that the lower denomination *(adha-ch-chheda)* is 360 days, and the result *(phala)* is given as $\frac{1807}{240}$ (i. e. $\frac{2168400 \cdot 727}{727 \cdot 800 \cdot 360}$).

In connection with this question of the age of the Bakhshālī work, I may note a circumstance which appears to point to a peculiar connection of it with the Brahmasiddhānta of Brahmagupta. There is a curious resemblance between the 50th sūtra of the Bakhshālī arithmetic, or rather with the algebraical example occurring in that sūtra, and the 49th sūtra of the chapter on algebra in the Brahmasiddhānta. In that sūtra, Brahmagupta first quotes a rule in prose, and then adds another version of it in the Āryā measure. Unfortunately the rule is not preserved in the Bakhshālī MS., but as in the case of all other rules, it would have been in the form of a shloka and in the North-western Prākrit (or 'Gāthā dialect'). Brahmagupta in quoting it, would naturally put it in what he considered correct Sanskrit prose, and would then give his own version of it in his favourite Āryā measure. I believe it is generally admitted that Indian arithmetic and algebra, at least, is of entirely native origin. While siddhānta writers, like Brahmagupta and his predecessor Aryabhaṭṭa, might have borrowed their astronomical elements from the Greeks or from books founded themselves on Greek science, they took their arithmetic from native Indian sources. Of the Jains it is well known that they possess astronomical books of a very ancient type, showing no traces of western or Greek

influence. In India arithmetic and algebra are usually treated as portions of works on astronomy. In any case it is impossible that the Jains should not have possessed their own treatises on arithmetic when they possessed such on astronomy. The early Buddhists, too, are known to have been proficients in mathematics. The prevalence of Buddhism in Northwestern India, in the early centuries of our era, is a well known fact. That in those early times there were also large Jain communities in those regions is testified by the remnants of Jain sculpture found near Mathurā and elsewhere. From the fact of the general use of the Northwestern Prākrit (or the 'Gāthā dialect') for literary purposes among the early Buddhists it may reasonably be concluded that its use prevailed also among the Jains between whom and the Buddhists there was so much similarity of manners and customs. There is also a diffusedness in the mode of composition of the Bakhshālī work which reminds one of the similar characteristic observed in Buddhist and Jain literature. All these circumstances put together seem to render it probable that in the Bakhshālī MS. we have preserved to us a fragment of an early Buddhist or Jain work on arithmetic (perhaps a portion of a larger work on astronomy) which may have been one of the sources from which the later Indian astronomers took their arithmetical information. These earlier sources, as we know, were written in the shloka measure, and when they belonged to the Buddhist or Jain literature, must have been composed in the ancient Northwestern Prākrit. Both these points are characteristics of the Bakhshālī work. I may add that one of the reasons why the earlier works were, as we are told by tradition, revised and rewritten in the Āryā measure by later writers such as Brahmagupta, may have been that in their time the literary form (Gāthā dialect) of the Northwestern Prākrit had come to be looked upon as a barbarous and ungrammatical jargon as compared with their own classical Sanskrit. In any case the Buddhist or Jain character of the Bakhshālī arithmetic would be a further mark of its high antiquity.

Throughout the Bakhshālī arithmetic the decimal system of notation is employed. This system rests on the principle of

the 'value of position' of the numbers. It is certain that this principle was known in India as early as 500 A. D. There is no good reason why it should not have been discovered there considerably earlier. In fact, if the antiquity of the Bakhshālī arithmetic be admitted on other grounds, it affords evidence of an earlier date of the discovery of that principle. As regards the *zero*, in its modern sense of a 'mark of position' and one of the ten fundamental figures of the decimal system (0 1 2 3 4 5 6 7 8 9), its discovery is undoubtedly much later than the discovery of the 'value of position'. It is quite certain, however, that the application of the latter principle to numbers in ordinary writing would have been nearly impossible without the employment of some kind of 'mark of position', or some mark to indicate the 'empty place' *(shūnya)*. Thus the figure 7 may mean either 'seven' or 'seventy' or 'seven hundred' according as it be or be not supposed to be preceded by one (7 · or 70) or two (7 · · or 700) 'empty places'. Unless the presence of these 'empty places' or the 'position' of the figure 7 be indicated, it would be impossible to read its 'value' correctly. Now what the Indians did, and indeed still do, was simply to use for this purpose the sign which they were in the habit of using for the purpose of indicating *any* empty place or omission whatsoever in a written composition; that is the dot. It seems obvious from the exigencies of writing that the use of the well known dot as the mark of an empty place must have suggested itself to the Indians as soon as they began to employ their discovery of the principle of 'value position' in ordinary writing. In India the use of the dot as a substitute of the zero must have long preceded the discovery of the proper zero, and must have been contemporaneous with the discovery of that principle. There is nothing in the Bakhshālī arithmetic to show that the dot is used as a proper zero, and that it is any thing more than the ordinary 'mark of an empty place'. The employment, therefore, of the decimal system of notation, such as it is, in the Bakhshālī arithmetic is quite consistent with the suggested antiquity of it.

I have already stated that the Bakhshālī arithmetic is written in tho so-called 'Gāthā dialect', or in that literary form of the Northwestern Prākrit, which preceded the employment, in secular composition, of the classical Sanskrit. Its literary form consisted in what may be called (from the Sanskrit point of view) an imperfect sanskritisation of the vernacular Prākrit. Hence it exhibits at every turn the peculiar characteristics of the underlying vernacular. The following are some specimens of orthographical peculiarities.

Insertion of euphonic consonants: of *m,* in *eka-m-ekatvaṁ, bhṛitako-m-ekapaṇḍitaḥ;* of *r,* in *tṛi-r-āshīti, labhate-r-aṣṭau.*

Insertion of *s: vibhaktaṁ-s-uttare, Kṣiyate-s-traya.* This is a peculiarity not elsewhere known to me, either in Prākrit or in Pāli.

Doubling of consonants: in compounds, *prathama-d-dhānte, eka-s-saṁkhyā;* in sentences, *yadi-ṣ-ṣaḍbhi, ete-s-samadhanā.*

Peculiar spellings: *tṛiṅshā* or *tṛiṅsha* for *triṁshat.* The spelling with the guttural nasal before *sh* occurs only in this word; e. q., *chatvāliṁsha* 40. Again *ṛi* for *ri* in *tṛidine, kṛiyate, vimishṛitaṁ, kṛiṇāti;* and *ri* for *ṛi* in *riṇaṁ, drisṭaḥ.* Again *katthyatāṁ* for *kathyatāṁ.* Again the *jihvāmūlīyo* and the *upadhmānīya* are always used before gutturals and palatals respectively.

Irregular sandhi: *ko so rā°* for *kaḥ sa rā°, dvayo kechi* for *dvayaḥ k°, dvayo cha* for *dvayash cha, dvibhi kri°* for *dvibhiḥ kri°, ādyo vi°* for *ādyor vi°, vivaritāsti* for *vivaritam asti.*

Confusion of the sibilants: *sh* for *ṣ,* in *shaṣṭi* 60, *māshako; ṣ* for *sh,* in *dashāṃṣha, viṣodhayet, ṣeṣaṁ; sh* for *s,* in *sāshyaṁ, sāsyatāṁ; s* for *sh,* in *esa* ‚this‘.

Confusion of *ṇ* and *n: utpaṇṇa.*

Elision of a final consonant: *bhājaye, kechi* for *bhājayet, kechit.*

Interpolation of *r: hrīṇaṁ* for *hīnaṁ.*

The following are specimens of etymological and syntactical peculiarities.

Absence of inflection: nom. sing. masc., *esha sā rāshi* for *rāshiḥ* (s. 50), *gavāṁ visheṣa kartavyaṁ* for *visheṣaḥ* (s. 51). Nom. plur., *sevya santi* for *sevyāḥ* (s. 53). Acc. plur., *dīnāra dattavān* for *dīnārān* (s. 53).

Peculiar inflection: gen. sing., *gatisya* for *gateḥ* (s. 15); ātm. for parasm., *ārjayate* for *arjayati* 'he earns' (s. 53); parasm. for ātm., *vikriṇāti* for *vikrīṇīte* 'he sells' (s. 54).

Change of gender: masc. for neut., *mūlā* for *mūlāni* (s. 55); neut. for masc., *vargaṁ* for *vargaḥ* (s. 50); neut. for fem., *yutiṁ cha kartavyā* for *yutish* (s. 50).

Exchange of numbers: plur. for sing., *(bhavet) lābhāḥ* for *lābhaḥ* (s. 54).

Exchange of cases: acc. for nom., *dvitīyaṁ paṁchadivase rasam ārjayate* for *dvitīyaḥ* (s. 53); acc. for instr., *kṣayaṁ saṁguṇya* for *kṣayeṇa* (s. 27); acc. for loc., *kiṁ kālaṁ* for *kasmin kāle* (s. 52); instr. for loc., *anena kālena* for *asmin kāle* (s. 53); instr. for nom., *prathamena dattavān* for *prathamo* (s. 53), or *ekena yāti* for *eko* (s. 15); loc. for instr., *prathame dattā* for *prathamena* (s. 53), or *mānave gṛihītaṁ* for *mānavena* (s. 57); gen. for dat., *dvitīyasya dattā* for *dvitīyāya* (s. 53).

Abnormal concord: incongruent cases, *ayaṁ praṣṭe* for *asmin* (s. 52); incongruent numbers, *esha lābhāḥ* for *lābhaḥ* (s. 54) *rājaputro kechi* for *rājaputrāḥ* (s. 53); incongruent genders, *sā kālaṁ* for *tat kālaṁ* (s. 52), *visheṣa kartavyaṁ* for *kartavyaḥ* (s. 51), *sā rāshiḥ* for *sa* (s. 50), *kāryaṁ sthitaḥ* for *sthitaṁ* (s. 14).

Peculiar forms: *nivarita* for *nivṛita, ārja* for *ārjana, divaddha* 'one and one-half', *chatvāliṁsha* 40, *paṁchāshama* 50th, *chaupaṁchāshama* 54th, *chaturāshīti* 84, *tṛi-r-āshīti* 83, etc.

The following extracts may serve as specimens of the text.

Sūtraṁ |

Ādyor visheṣadviguṇaṁ chayashuddhi vibhājitaṁ |
Rūpā*dhikaṁ tathā kā*laṁ gatisāsyaṁ tadā bhavet ‖

tadā ‖

Dvayāditṛichayash chaiva dvi*chayatryā*dikottaraḥ |
Dvayo cha bhavate paṁthā kena kālena sāsyatāṁ ‖

sthāpanaṁ kṛiyate | *eṣāṁ* ‖ ā $\frac{2}{1}$ ‖ u $\frac{3}{1}$ ‖ pa $\frac{\cdot}{1}$ ‖ dvi ‖ ā $\frac{3}{1}$ ‖ u $\frac{2}{1}$ ‖ pa $\frac{\cdot}{1}$ ‖ karaṇaṁ ‖ ādyor visheṣa . . . . tā dvi 2 . . . .

tadā ‖

. . . . ‖
. . . . ‖

ā $\frac{5}{1}$	u $\frac{6}{1}$	pa $\frac{\cdot}{1}$	dha $\frac{\cdot}{1}$
ā $\frac{10}{1}$	u $\frac{3}{1}$	pa $\frac{\cdot}{1}$	dha $\frac{\cdot}{1}$

karaṇaṁ | ādyor visheṣaṁ ādi *5* | *10 visheṣa* 5 | chayashuddhi chayaṁ 6 | 3 shuddhi 3 ādisheṣa 5 dviguṇaṁ 10 uttaravisheṣa 3 vibhaktaṁ $\frac{10}{3}$ sarūpaṁ $\frac{13}{3}$ eṣa padaṁ anena kā*lena* samadhanā bhavanti ‖ pratyayaṁ ‖ rūpoṇakaraṇena phalaṁ ‖ $\frac{65}{\text{dvi}}$ 65 ‖ ‖ Aṣṭhādasashamasūtraṁ 18 ‖ ⁜ ‖

Idānī*ṁ* suvarṇakṣayaṁ vakṣyā*mi* yasyeda*ṁ* sūtraṁ |

Sūtraṁ |

Kṣayaṁ saṁguṇya kanakās tadyuti-b-bhājay*et* tataḥ |
Saṁyutair eva kanakair ekaikasya kṣayo hi saḥ ‖

tadā ‖

*Ekadvitrichatu*ssaṁkhya suvarṇā māṣakai riṇai |
Ekadvitrichatussaṁkh*yai* rahitā samabhāgatāṁ ‖

sthāpanaṁ kṛiyate | eṣāṁ ‖ $\frac{1+}{1}$ ‖ $\frac{2+}{2}$ ‖ $\frac{3+}{3}$ ‖ $\frac{4+}{4}$ ‖ karaṇaṁ ‖ kṣayaṁ saṁguṇya kanakādibhi kṣayena saṁguṇya jātaṁ | 1 | 4 | 9 | 16 | *tadyuti* | eṣa yati 30 kanakā yuti 10 anena bhaktvā labdhaṁ . . . .

. . . .	$\frac{10}{1}$	$\frac{30}{1}$	$\frac{1}{1}$	*pha māse* $\frac{3}{1}$
. . . .	$\frac{10}{1}$	$\frac{30}{1}$	$\frac{2}{1}$	*pha māse* $\frac{6}{1}$
. . . .	$\frac{10}{1}$	$\frac{30}{1}$	$\frac{3}{1}$	*pha māse* $\frac{9}{1}$
. . . .	$\frac{10}{1}$	$\frac{30}{1}$	$\frac{3}{1}$	pha māse $\frac{12}{1}$

tadā ‖

Ekadvitṛ*chatussaṁkhyā* suvarṇa projjhitā ime |
Māsakā dvitṛitāṁ chaiva chatuḥpaṁchakarāṁshakaiṁ[1]) kim kṣayaṁ ‖

‖ $\begin{matrix}1\\1\\2\end{matrix}$ ‖ $\begin{matrix}2\\1\\3\end{matrix}$ ‖ $\begin{matrix}3\\1\\4\end{matrix}$ ‖ $\begin{matrix}4\\1\\5\end{matrix}$ ‖ karaṇaṁ | kṣayaṁ saṁguṇya kanakā esha sthāpyate | | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | -s-tadyuti-b-bhājayeta[2]) tataḥ harasāsye kṛite yutaṁ | $\frac{163}{60}$ | saṁyutaiḥ kanakair bhaktvā tadā kanakā 10 anena bhaktaṁ jātaṁ | $\frac{163}{60}$ | esha ekaikasuvarṇasya kṣayaṁ ‖ pra*tyayaṁ trairā*shikena kartavya ‖

$\frac{10}{1}$	$\frac{163}{60}$	$\frac{1}{1}$	*pha* $\frac{163}{600}$	. . . .
$\frac{10}{1}$	$\frac{163}{60}$	$\frac{2}{1}$	*pha* $\frac{163}{600}$	. . . .
$\frac{10}{1}$	$\frac{163}{60}$	$\frac{3}{1}$	*pha* $\frac{163}{600}$	. . . .
$\frac{10}{1}$	$\frac{163}{60}$	$\frac{4}{1}$	pha $\frac{163}{600}$	. . . .

tadā ‖

. . . . |
. . . . shruṇushva me ‖
Krameṇa dvaya māṣādi uttare ekahīnatāṁ |
Suvarṇaṁ me *tu* sammishrya katthyatām gaṇakottama ‖

sthāpanaṁ ‖ $\frac{4+}{5}$ ‖ $\frac{5+}{6}$ ‖ $\frac{6+}{7}$ ‖ $\frac{7+}{8}$ ‖ $\frac{8+}{9}$ ‖ $\frac{9+}{10}$ ‖ $\frac{1+}{2}$ ‖ $\frac{2+}{3}$ ‖ $\frac{3+}{4}$ ‖ kṣayaṁ saṁguṇya jātaṁ 20 | 30 | 42 | 56 | 72 | 90 | 2 | 6 |[3]) eṣāṁ yuti 330 kana*kā*naṁ yuti 45 | anena bhaktvā labdhaṁ | $\frac{330}{45}$ | paṁchadash abhāge-sh-chheda kṛiyate | phalaṁ | 7 she $\frac{1}{3}$ | esha ekaikamāshakakṣayaṁ | pratyaya trairā*shikena* | $\frac{45}{1}$ | $\frac{330}{1}$ | $\frac{1}{1}$ | phalaṁ $\frac{22}{3}$ | evaṁ sarveṣāṁ pratyaya *kartavya* ‖ . . . .
. . . .

Saptaviṁshatiṁasūtraṁ 27 ‖ ⁜ ‖

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[1]) Read *chatuḥpaṁchāṁshaṁ kiṁ kṣayaṁ*, metri causa.
[2]) Read *bhājayet.*
[3]) Here | 12 | is omitted in the text, by mistake.

Sūtraṁ |

Ahadravyaharāshauta[1]) tadvisheṣaṁ vibh*ājayet* |
Yallabdhaṁ dviguṇaṁ kālaṁ dattā samadhanā prati ‖
tadā ‖
Tṛidine ārjaye paṁcha bhṛitako-m-ekapaṇḍitaḥ |
Dvitīyaṁ paṁcha*di*vase rasam ārjayate budhaḥ ‖
Prathamena dvitīyasya sapta dattāni . . taḥ |
Datvā samadhanā jātā kena kālena katthyatāṁ ‖

‖ $\frac{5}{3}$ *rū* ‖ $\frac{6}{5}$ *rū* ‖ . . . ṁ . . . . ṁ h*arā*ṁshauta *tadvisheṣa*ṁ .

. . . .

. . . .

anena kālena samadhanā bhavanti ‖ pratyaya trairāshike kṛiyate

$\frac{3}{1}$	$\frac{5}{1}$	$\frac{30}{1}$	pha 50
$\frac{5}{1}$	$\frac{6}{1}$	$\frac{30}{1}$	pha 36

prathame dvitīyasya-s-sapta dattā | 7 sheṣaṁ 43 ‖ 43 | 43 ete samadhanā jātā ‖
tadā ‖
*Rā*japutro dvayo kechi nṛipati-s-sevya santi vaiḥ |
M-ekāsyāhne dvaya-ṣ-ṣadbhāgā[2]) dvitīyasya divarddhakaṁ ‖
Prathamena dvitīyasya ḍasha dīnāra dattavān |
Kena kālena samatāṁ gaṇayitvā vadāshu me ‖

‖ $\frac{13}{6}$ ‖ $\frac{3}{2}$ ‖ dattaṁ $\frac{10}{1}$ ‖ karaṇaṁ ‖ aha dravyavisheṣaṁ cha | tatra

. . . .

. . . .

pratyayaṁ trairāshikena dvitīyasya 10 dattā jātā | 55

$\frac{1}{1}$	$\frac{13}{6}$	$\frac{30}{1}$	pha 65
$\frac{1}{1}$	$\frac{3}{2}$	$\frac{30}{1}$	pha 45

prathamena dvi- | 55 ‖ samadhanā jātā ‖ Sūtraṁ tṛipaṁchāshamaḥ sūtrāṁ 53 ‖ ⁜ ‖

Translation.

The 18th Sūtra.

Let twice the difference of the two initial terms be divided by the difference of the (two) increments. The result augmented by one shall be the time that determines the progression.

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[1]) Read °*harāṁshauta.*

[2]) Read *ekasyāhne dviṣaḍbhāgā.* The error appears to have been noticed by the scribe of the manuscript.

First Example.

A person has an initial (speed) of two and an increment of three, another has an increment of two and an initial (speed) of three. Let it now be determined in what time the two persons will meet in their journey.

The statement is as follows:

N° I, init. term 2, increment 3, period x
N° II, » » 3, » 2, » x.

Solution: the difference of the two initial terms (2 and 3 is 1; the difference of the two increments 3 and 2 is 1; twice the difference of the initial terms 1 is 2, and this, divided by the difference of the increments 1 is $^2/_1$, and augmented by 1 is $^3/_1$; this is the period. In this time [3] they meet in their journey which is 15).

Second Example.

(The problem in words is wanting; it would be something to this effect: A earns 5 on the first and 6 more on every following day; B earns 10 on the first and 3 more on every following day; when will both have earned an equal amount?)

Statement:

N° 1, init. term 5, increment 6, period x, possession x
N° 2, » » 10, » 3, » x, » x.

Solution: 'Let twice the difference of the two initial terms', etc.; the initial terms are 5 and 10, their difference is 5. 'By the difference of the (two) increments'; the increments are 6 and 3; their difference is 3. The difference of the initial terms 5, being doubled, is 10, and divided by the difference of the increments 3, is $\frac{10}{3}$, and augmented by one is $\frac{13}{3}$. This (i. e. $\frac{13}{3}$ or $4\frac{1}{3}$) is the period; in that time the two persons become possessed of the some amount of wealth.

Proof: by the '*rūpoṇa*' method the sum of either progression is found to be 65 (i. e., each of the two persons earns 65 in $4^1/_3$ days).

The 27th Sūtra.

Now I shall discuss the wastage (in the working) of gold, the rule about which is the following.

Sūtra.

Multiplying severally the parts of gold with the wastage, let the total wastage be divided by the sum of the parts of gold. The result is the wastage of each part (of the whole mass) of gold.

First Example.

Suvarṇas numbering respectively one, two, three, four are subject to a wastage of *māṣakas* numbering respectively one, two, three, four. Irrespective of such wastage they suffer an equal distribution of wastage. (What is the latter?)

The statement is as follows:

Wastage	— 1,	— 2,	— 3,	— 4 *māṣaka*
Gold	1,	2,	3,	4 *suvarṇa.*

Solution: 'Multiplying severally the parts of gold with the wastage', etc.; by multiplying with the wastage, the product 1, 4, 9, 16 is obtained; 'let the total wastage', its sum is 30; the sum of the parts of gold is 10; dividing with it, we obtain 3. (This is the wastage of each part, or the average wastage, of the whole mass of gold.)

(Proof by the rule of three is the following:) as the sum of gold 10 is to the total wastage of 30 *maṣakas,* so the sum of gold 4 is to the wastage of 12 *maṣakas,* etc.

Second Example.

There are *suvarṇas* numbering one, two, three, four. There are thrown out the following *māṣakas*: one-half, one-third, one-fourth, one-fifth. What is the (average) wastage (in the whole mass of gold)?

Statement:

quantities of gold,	1,	2,	3,	4 *suvarṇas*
wastage	$^1/_2$,	$^1/_3$,	$^1/_4$,	$^1/_5$ *māṣakas.*

Solution: 'Multiplying severally the parts of gold with the wastage', the products may thus be stated: $^1/_2$, $^2/_3$, $^3/_4$, $^4/_5$. 'Let

the total wastage be divided'; the division being directed to be made, the total wastage is $\frac{163}{60}$; dividing 'by the sum of the parts of gold'; here the sum of the parts of gold is 10; being divided by this, the result is $\frac{163}{600}$. This is the wastage of each part of the whole mass of gold.

Proof: may be made by the rule of three: as the sum of the parts of gold 10 is to the total wastage of $\frac{163}{60}$ *maṣaka*, so the sum of gold 4 is to the wastage of $\frac{163}{600}$ *maṣaka*, etc.

Third Example.

(The problem in words is only partially preserved, but from its statement in figures and the subsequent explanation, its purport may be thus restored.)

Of gold *māṣakas* numbering respectively five, six, seven, eight, nine, ten, quantities numbering respectively four, five, six, seven, eight, nine, are wasted. Of another metal numbering in order two *māṣaka*, etc. (i. e., two, three, four) also quantities numbering in order one etc. (i. e. one, two, three) are wasted. Mixing the gold with the alloy, O best of arithmeticians, tell me (what is the average wastage of the whole mass of gold)?

Statement:

wastage:	— 4,	— 5,	— 6,	— 7,	— 8,	— 9;	— 1,	— 2,	— 3,
gold:	5,	6,	7,	8,	9,	10;	2,	3,	4.

(Solution:) 'Multiplying severally the parts of gold with the wastage', the product is 20, 30, 42, 56, 72, 90, 2, 6, 12; their sum is 330; the sum of the parts of gold is 45; dividing by this we obtain $\frac{330}{45}$; this is reduced by 15 (i. e. $\frac{22}{3}$); the result is 7 leaving $^1/_3$ (i. e. $7^1/_3$); that is the wastage of each *māṣaka* (of mixed gold).

Proof: by the rule of three: as the total gold 45 is to the total wastage 330, so 1 *māṣaka* of gold is to $\frac{22}{3}$ parts of wastage. In the same way the proof of all (the other) items is to be made (i. e., $45 : 330 = 5 : \frac{110}{3}$; $45 : 330 = 6 : 44$; $45 : 330 = 7 : \frac{154}{3}$; $45 : 330 = 8 : \frac{176}{3}$; $45 : 330 = 9 : 66$; $45 : 330 = 10 : \frac{220}{3}$).

The 53[d] sūtra.

Let the portion given from the daily earnings be divided by the difference of the latter. The quotient, being doubled, is the time (in which), through the gift, their possessions become equal.

First Example.

Let one serving pandit earn five in three days; another learned man earns six in five days. The first gives seven to the second from his earnings; having given it, their possessions become equal; say, in what time (this takes place)?

Statement N° 1, $\frac{5}{3}$ earnings of 1 day, N° 2, $\frac{6}{5}$ earnings of 1 day; gift 7.

Solution: 'Let the portion of the daily earnings be divided by the difference of the latter'; (here the daily earnings are $\frac{5}{3}$ and $\frac{6}{5}$; their difference is $^7/_{15}$; the gift is 7; divided by the difference of the daily earnings $^7/_{15}$, the result is 15; being doubled, it is 30; this is the time), in which their possessions become equal.

Proof: may be made by the rule of three: $3:5 = 30:50$ and $5:6 = 30:36$; 'the first gives seven to the second' 7, remainder 43; hence 43 and 43 are their equal possessions.

Second Example.

Two Rājpūts are the servants of a king. The wages of one per day are two and one-sixth, of the other one and one-half. The first gives to the second ten *dīnārs*. Calculate and tell me quickly, in what time there will be equality (in their possessions)?

Statement: daily wages $\frac{13}{6}$ and $\frac{3}{2}$; gift 10.

Solution: 'and the daily earnings'; here (the daily earnings are $\frac{13}{6}$ and $\frac{3}{2}$; their difference is $\frac{2}{3}$; the gift is 10; divided by the difference of the daily earnings $\frac{2}{3}$, the result is 15; being doubled, it is 30. This is the time, in which their possessions become equal).

Proof by the rule of three: $1 : \frac{13}{6} = 30 : 65$; and $1 : \frac{3}{2} = 30:55$. The first gives 10 to the second; hence 55 and 55 are their equal possessions.

Notes.

1. In the text, the italicised words are conjecturally restored portions. The dots signify the number of syllables *(akṣara)* which are wanting in the manuscript. The serpentine lines indicate the lines lost at the top and bottom of the leaves of the manuscript. In the translation the bracketed portions supply lost portions of the manuscript. The latter can, to a great extent, be restored by a comparison of the several examples. Occasionally words are added in brackets to facilitate the understanding of the passage.

2. Sūtra 18. Problems on progression. Two persons advance from the same point. At starting B has the advantage over A; but afterwards A advances at a quicker rate than B. Question: when will they have made an equal distance? In other words, that period of the two progressions is to be found, where their sums coincide. The first example is taken from the case of two persons travelling. B makes 3 miles on the first day against 2 miles of A; but A makes 3 miles more on each succeding day against B's 2 miles. The result is that at the end of the 3d day they meet, after each has travelled 15 miles. For A travels $2 + (2 + 3) + (2 + 3 + 3) = 15$ miles, and B $3 + (3 + 2) + (3 + 2 + 2) = 15$ miles. The second example is taken from the case of two traders. At starting B has the advantage of possessing 10 *dīnārs* against the 5 of A; but in the sequel A gains 6 *dīnārs* more on each day against the 3 of B. The result is that after $4^1/_3$ days, they possess an equal amount of *dīnārs,* viz. 65.

3. Sūtra 27. Problems on averages *(samabhāgatā).* Certain quantities of gold suffer loss at different rates. Question: what is the average loss of the whole? The first problem is very concisely expressed; the question is understood; some words, like *kutogatā,* must be supplied to *samabhāgatāṁ.*
